

# The Unitary Mechanism of Infrared Freezing in QCD with massive gluons

D.V.Shirkov

*Bogoliubov Theor. Lab, JINR, Dubna, 141980, Russia*  
*shirkovd@thsun1.jinr.ru*

## Abstract

A “natural” model for the QCD invariant (running) coupling, free of the IR singularity, is proposed. It is based upon the hypothesis of finite gluon mass  $m_{gl}$  existence and, technically, uses an accurate treating of threshold behavior of Feynman diagram contribution. The model correlates with the unitarity condition.

Quantitative estimate, performed in the one-loop approximation yields a reasonable lower bound for this mass  $m_{gl} > 150$  MeV and a smooth IR freezing at the level  $\alpha_s(Q^2) \lesssim 1$ .

## 1 Introduction

The issue of the infrared (IR) behavior of the strong interaction becomes more and more actual from the physical point of view along with the further experimental data accumulation. In the perturbative quantum chromodynamics (pQCD) this behavior is burdened with “unphysical” singularities marked with the so called scale parameter  $\Lambda \simeq 300$  MeV. These singularities contradict some general principles of the local QTF. In the “small momentum transfer region”  $Q \lesssim 3\Lambda$  they violate the weak coupling regime and complicate theoretical interpretation of data.

Quite recently attempts have been made to devise models for invariant (running) coupling  $\alpha_s(Q^2)$  and for observables free of singularities in the IR region. One idea was to exploit the scheme arbitrariness of the third beta-function coefficient  $\beta_3$ . Performing an “optimization” of the beta-function the authors of paper [1] argued for special solution with an IR fixed point and  $\alpha_s(Q^2) < 1$  bounded in the whole region  $Q^2 \leq 1\text{GeV}^2$ . In the paper [2] the allowance was made for nonperturbative contributions into  $\alpha_s(Q^2)$  emerged from the vacuum QCD background fields. Here, the adjusting parameter, the background (hybrid) gluon mass  $M_B$ , enters as an IR regulator  $\ln Q^2 \rightarrow \ln(Q^2 + M_B^2)$  of the gluonic logs. At  $M_B = 1.5$  GeV the running coupling maximum value turned out to be  $\alpha_s(0) \simeq 0.4$ . One more trick [3] uses an imperative of the Källén–Lehmann analyticity in the complex  $Q^2$  plane. Effectively, it results in the smooth freezing of  $\alpha_s(Q^2)$  at the level of  $0.5 - 0.7$  for  $0.1\text{ GeV}^2 \leq Q^2 \leq 1\text{ GeV}^2$  and in the universal, i.e., loop-independent IR limiting value  $\alpha_s^{max} = \alpha_s(0) = 4\pi/\beta_1 \simeq 1.4$ .

In the last, so called “invariant analytic”, approach (for a recent review, see [4]) the finiteness and smoothness of the  $\alpha_s(Q^2)$  behavior in the IR region is achieved only due to the analyticity imperative, without introducing any adjustable parameters. Qualitatively, the effect of smooth freezing arises here due to the addition of a power (in the  $Q^2$  variable) and nonanalytic (in the coupling constant  $\alpha_s$ ), i.e., nonperturbative terms which restore the Källén–Lehmann analyticity for renormalization – invariant quantities.

In this note, we consider one more possibility of constructing the invariant coupling  $\alpha_s(Q^2)$  that is free of unphysical singularities and *does not involve explicit nonperturbative contributions*. Here, the  $Q^2$  power contributions appear due to threshold effects and an essential physical ingredient is the assumption of the **finite gluon mass existence**.

Our model expression for  $\alpha_s(Q^2)$  is obtained by the renormalization group (RG) summation of mass-dependent one-loop Feynman diagrams contribution — see, below, Eqs. (2) and (3). It depends upon the gluonic  $m_{\text{gl}}$  and light quark  $m_{\text{u,d,s}}$  masses, in the IR region  $Q^2 > 0$  obeys a nonsingular behavior with a finite limiting value  $\alpha_s(0)$ , and as  $Q^2/m^2 \rightarrow \infty$  smoothly transits into the usual asymptotic freedom formula.

## 2 Massive loops

Our starting point is quite simple and natural — we propose to take into the account the threshold mass dependence while considering the infrared region. As it is well known, the “leading UV logs”, which after the RG summation yield, in particular, the Landau pole, arise from the one-loop Feynman diagrams. However, in the IR region these diagrams behavior, generally, is far from being logarithmic. For instance, to the virtual dissociation of a vector particle (photon, gluon) into a massive fermion-antifermion pair ( $e^+ + e^-$ ;  $q + \bar{q}$ ) in the  $s$ -wave state, there corresponds a function  $I_s(Q^2/M^2)$  that can be represented in the form of a spectral integral

$$I_s(z) = z \int_1^\infty \frac{k_s(\sigma) d\sigma}{\sigma(\sigma + z)}; \quad k_s(\sigma) = \sqrt{\frac{\sigma - 1}{\sigma}} \left(1 + \frac{1}{2\sigma}\right) \quad (1)$$

and in the space-like region  $z > 0$  is a positive, monotonically growing function with logarithmic asymptotic behavior  $I_s(z) \simeq \ln z - C_s + O(z^{-1})$ ;  $C_s = 5/3$ . For definiteness and in accordance with the QED tradition we have subtracted it at  $z = 0$ . This agreement turns out to be convenient for our purpose in the “low  $Q^2$  region”.

In QCD, besides  $I_s(z)$ , essential is a function  $I_p(z)$  describing a virtual dissociation of a vector gluon into a pair of massive vector or scalar particles (gluons or “ghosts”) in the  $p$ -wave state. This function can also be represented in the form (1) with an adequate weight function  $k_p(\sigma)$  and obeys the same simple properties at  $z > 0$ . In particular, in the UV limit  $I_p(z) \simeq \ln z - C_p + O(1/z)$ . In our analysis only asymptotic constant  $C_p$  will be of importance. It relates to the integral over the phase volume. Due to this  $C_p > C_s$ . For the qualitative estimate we use [5]  $C_p = 8/3$ .

Here, we assume that a virtual gluon has a finite (reasonably small — see below) mass  $m_{\text{gl}}$ . We postpone for the future any detailed discussion of a possible origin of this mass noting that the mechanism should be based upon a deeper understanding of the ground state structure of the quantum gauge  $SU(3)$  field. As a provisional *ad hoc* working model one could imply the picture of spontaneous symmetry breaking, analogous to the  $SU(2)$

case (for a fresh discussion of the subject, see, e.g., Ref.[6]). In addition, one should have in mind the upper phenomenological bound

$$m_{\text{gl}} \lesssim 600 \text{ MeV} ,$$

corresponding to the absence of a direct experimental signal for this mass existence.

### 3 Massive Renorm–group

For an analysis of the invariant QCD coupling at small space-like values of the variable  $Q^2 < \Lambda^2$  we use the “massive”, that is mass dependent, renormalization group as it has been explicitly formulated in the pioneer papers [7] in the mid-fifties.

In particular, we shall exploit the fact that massive RG, quite in parallel to the widely used massless one, sums all iterations of a one-loop contribution in the invariant coupling

$$a_s(Q^2)_{\text{pert}} = a_s - a_s^2 A_1(Q^2, m^2) + a_s^3 (A_1(Q^2, m^2))^2 + \dots$$

into the geometric progression [8]

$$a_s(Q^2)_{\text{rg},1} = \frac{a_s}{1 + a_s A_1(Q^2, m^2)} . \quad (2)$$

Here, we use the notation for the so-called *couplant*

$$a_s(Q^2) = \alpha_s(Q^2)/4\pi , \quad a_s = \alpha_s/4\pi .$$

Analogous (approximate) RG summed mass–dependent expressions are known [9] for the two-loop case — see below Eqs. (5) and (6) — as well.

Let us make a comment on the renormalization (subtraction) scheme (RS). We use the subtraction at  $Q^2 = 0$ , that is the MOM–scheme instead of the massless  $\overline{\text{MS}}$  one which is in common practice. Among QCD practitioners there exists a strong prejudice against any MOM–schemes due to their gauge dependence. For our analysis it is essential to use mass-dependent expressions for the diagram contributions. The particular RS is not of principal importance. One could transit from our scheme to a  $\overline{\text{MS}}$  scheme by standard rules. In particular, connection between MOM, massive  $\overline{\text{MS}}$  and the popular massless  $\overline{\text{MS}}$  schemes has been discussed in detail in papers [9, 10] and we have no possibility of repeating the discussion in this short note.

### 4 The one-loop analysis

For the one-loop contribution to an invariant couplant we use the expression<sup>1</sup>

$$A_1(Q^2, m^2) = 11 I_s \left( \frac{Q^2}{m_{\text{gl}}^2} \right) - \frac{2}{3} \sum_q I_p \left( \frac{Q^2}{m_q^2} \right) . \quad (3)$$

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<sup>1</sup> Generally, the  $A_1(Q^2, m^2)$  should combine several (propagator and vertex) contributions. However, for our semiquantitative analysis only UV behaviour with asymptotic constants  $C_{s,p}$  will be essential.

This expression by convention turns to zero at  $Q^2 = 0$ . Hence, in (2)  $a_s = a_s(0)$ . In the opposite limit at  $Q^2 \gg m^2$ , we have

$$A_1(Q^2, m^2) \rightarrow 11 \left( \ln \frac{Q^2}{m_{\text{gl}}^2} - \frac{8}{3} \right) - \frac{2}{3} \sum_q \left( \ln \frac{Q^2}{m_q^2} - \frac{5}{3} \right).$$

In the three-quark region, this gives

$$A_1(Q^2, m^2) \simeq 9 \ln \frac{Q^2}{m_{\text{gl}}^2} - 2 \ln \frac{m_{\text{gl}}^2}{m_s^2} - \frac{4}{3} \ln \frac{m_s^2}{m_u m_d} - 26; \quad Q^2 \gg m_c^2, m_{\text{gl}}^2.$$

In the massless case to the expression  $1/a + A_1(Q^2, m^2)$  there corresponds  $9 \ln(Q^2/\Lambda^2)$ . Hence, we get the relation

$$\frac{1}{a_s} + 22 \ln \frac{m_s}{m_{\text{gl}}} + 18 \ln \frac{\Lambda}{m_s} = 26 + \frac{4}{3} \ln \frac{m_s^2}{m_u m_d}$$

between the combination  $1/a - 22 \ln m_{\text{gl}}$  and the QCD scale parameter  $\Lambda$ . For a quantitative estimate let us define an “effective one-loop QCD scale parameter” from the condition  $\alpha_s(M_\tau^2) = 0.37$  that yields  $\Lambda_1 = 250$  MeV.

Now putting  $m_u = 5$  MeV,  $m_d = 10$  MeV,  $m_s = 150$  MeV,  $m_{\text{gl}} = m_s/\sigma$ , we arrive at the relation

$$\frac{1}{a_s} = 25 - 22 \ln \sigma; \quad \alpha_s(0) = \frac{2\pi}{12.5 + 11 \ln(m_{\text{gl}}/m_s)}. \quad (4)$$

The most important qualitative result that follows from it consists in the existence of a reasonably small lower bound for the gluon mass corresponding to the  $\alpha_s(0) \simeq 1$  condition.

We have

$$100 \text{ MeV} < m_{\text{gl}} < 600 \text{ MeV}.$$

In the Table a few values of the IR limit  $\alpha_s(0)$  for some  $m_{\text{gl}}$  of this interval are given.

Table

$m_{\text{gl}}/\text{MeV}$	100	150	200	300	450
$\alpha_s(0)$	0.87	0.55	0.41	0.36	0.31

As it follows from the Table, there exists a rather wide interval of the gluonic mass  $m_{\text{gl}}$  with “reasonably small”  $\alpha_s(0) \lesssim 1$  values.

Let us note that the lower bound as obtained from relation (4) has only a qualitative nature. The point is that due to the relation  $m_{u,d} \ll m_{\text{gl}}$  the derivative  $da(x)/dx$  at  $x = 0$  is positive and in the region  $0 < Q^2 < m_{\text{gl}}^2$  the light quark contribution dominates. The real place of possible “blowing up” is close to  $Q_*^2 = 2m_{\text{gl}}^2$ . As a result, the real lower bound for  $m_{\text{gl}}$  corresponds to the condition  $a_s < 1/(28 - 4 \ln \sigma)$ , and turns out to be close to the strange quark mass.

The upper bound existence for the  $\alpha_s(0)$  value resembles us the property of unitary models for lower partial waves of hadron scattering in the low-energy elastic region. These models were popular in the sixties – see, e.g., Refs. [11]. Besides analyticity, they satisfy the two-particle unitarity condition. One-loop diagrams summed in the solution (2), (3) just relate to this last condition. This is the reason for associating our construction with unitarity.

## 5 Two-loop corrections

For a more accurate numerical description of the invariant coupling in the IR region one can use the two-loop massive perturbation expansion

$$a_s(Q^2)_{\text{pert},2} = a_s - a_s^2 A_1(Q^2, m^2) + a_s^3 A_1^2 - a_s^3 A_2(Q^2, m^2) + \dots \quad (5)$$

An approximate two-loop massive RG solution is of the form [9]

$$a_s(Q^2)_{\text{rg},2} = a_s \left\{ 1 + a_s A_1(Q^2, m^2) + a_s \frac{A_2(Q^2, m^2)}{A_1(Q^2, m^2)} \ln(1 + a_s A_1(Q^2, m^2)) \right\}^{-1} \quad (6)$$

At small  $a_s$  values this expression corresponds to (5). At the same time, at  $Q^2 \gg m^2$  it can be represented in the usual form

$$a_s^{-1}(Q^2)_{\text{rg},2} \rightarrow \beta_1 \left\{ \ln \frac{Q^2}{\Lambda^2} + b_1 \ln \left( \ln \frac{Q^2}{\Lambda^2} \right) \right\}; \quad b_1 = \frac{\beta_2}{\beta_1^2}.$$

As it follows from (6), in the IR region

$$\frac{1}{a_s(Q^2)} = \frac{1}{a_s} + A_1(Q^2, m^2) + a_s A_2(Q^2, m^2). \quad (7)$$

Hence, the two-loop contribution  $A_2(Q^2, m^2)$  is here suppressed by a small numerical factor  $a_s = \alpha_s(0)/4\pi \lesssim 1/10$  and cannot seriously influence the one-loop estimate obtained above.

## 6 Conclusion

The estimate obtained shows that an accurate description of the threshold effects related to the light quarks and, especially, to gluons allows us to formulate one more resolution of the issue of unphysical IR singularities in the pQCD. The price of this resolving consists in introducing of the only parameter – the gluon finite mass.

Thence, it is possible to keep the QCD invariant coupling  $\alpha_s(Q^2)$  in the weak coupling domain in the whole space-like region  $0 < Q^2 < \infty$  for the gluon mass values

$$200 \text{ MeV} < m_{\text{gl}} < 600 \text{ MeV}.$$

This theoretical “window” for the gluonic mass could be enlarged if we change the light quark masses from their current values to some effective ones  $m_{u,d}^{eff} \sim m_\pi$  [14]. Here, one can use the gluon effective mass of the same order of magnitude.

To conclude, let us remark that the present model, in reality, is not very far from our previous construction Refs.[3] with an explicit introducing of nonperturbative contribution. A gluon mass  $m_{gl}$ , being considered in the light of the genuine gauge-invariant QCD Lagrangian, certainly represents an effective nonperturbative parameter. The same is true for the abovementioned  $m_{u,d}^{eff}$ .

In our opinion, this demonstrates once more (compare, e.g., with discussion of the QED case in Ref.[15]) that the ghost-pole trouble is not a physical problem. It is a technical drawback inherent to usual, the pQCD one, way of theoretical analysis.

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